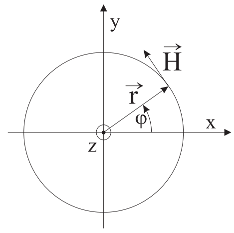
Theorie der Felder und Wellen 2	Name:
Blatt 17	Matrikelnummer:

Aufgabe 48

a)



$\oint \vec{H}\,\mathrm{d}\vec{s} = I$

- in Zylinderkoordinaten: $\vec{H} = \underbrace{\dfrac{I_0\cos(\omega_1 t)}{2\pi r}}_{H_\varphi(r)}\vec{e}_\varphi$

- in kartesischen Koordinaten: $\vec{H} = \frac{I_0 cos(\omega_1 t)}{2\pi r}\begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix} = \frac{I_0 cos(\omega_1 t)}{2\pi\sqrt{x^2+y^2}}\begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$

$\vec{A} = \frac{\mu_0}{4\pi}\iiint\limits_V \frac{\vec{J}(\vec{r}')\,\mathrm{d}V'}{|\vec{r}-\vec{r}'|} \qquad \rightarrow \qquad \vec{A} = \begin{pmatrix} 0 \\ 0 \\ A_z \end{pmatrix}$ (nur z-Komponente)

- zunächst Berechnung für einen endlich langen Leiter $(-z_0 \le z \le z_0)$:

$A_z = \frac{\mu_0}{4\pi}\int\limits_{-z_0}^{z_0} \frac{I_1\,\mathrm{d}z'}{\sqrt{x^2+y^2+(z-z')^2}}$ Substitution: $u := z - z'$, $\mathrm{d}u = -\,\mathrm{d}z'$

$A_z = \frac{\mu_0}{4\pi}\int\limits_{z+z_0}^{z-z_0} \frac{-I_1\,\mathrm{d}u}{\sqrt{x^2+y^2+u^2}} = \frac{\mu_0 I_1}{4\pi}\left[\ln(u+\sqrt{x^2+y^2+u^2})\right]_{u=z+z_0}^{z-z_0} = \frac{\mu_0 I_1}{4\pi}\ln\frac{z+z_0+\sqrt{x^2+y^2+(z+z_0)^2}}{z-z_0+\sqrt{x^2+y^2+(z-z_0)^2}}$

- Übergang zu einem unendlich langen Leiter $(z_0 \to \infty)$:

$A_z = \frac{\mu_0 I_1}{4\pi}\lim_{z_0\to\infty}\ln\frac{z+z_0+\sqrt{x^2+y^2+(z+z_0)^2}}{z-z_0+\sqrt{x^2+y^2+(z-z_0)^2}} = \frac{\mu_0 I_1}{4\pi}\lim_{z_0\to\infty}\ln\frac{z_0\left(1+\sqrt{\frac{x^2+y^2}{z_0^2}+1}\right)}{z_0\left(-1+\sqrt{\frac{x^2+y^2}{z_0^2}+1}\right)}$

$A_z = \frac{\mu_0 I_1}{4\pi}\lim_{z_0\to\infty}\ln\frac{1+1}{-1+\frac{x^2+y^2}{z_0^2}+1} = \frac{\mu_0 I_1}{4\pi}\lim_{z_0\to\infty}\ln\frac{4z_0^2}{x^2+y^2} = \frac{\mu_0 I_1}{4\pi}\lim_{z_0\to\infty}\ln\frac{4z_0^2}{r_0^2} + \ln\frac{r_0^2}{x^2+y^2}$ mit $r_0^2 = const. \neq 0$

- Problem: $A_z \to \infty$, da $\lim\limits_{z_0\to\infty}\ln\frac{4z_0^2}{r_0^2} \to \infty$

- Abhilfe: magnetisches Vektorpotential $\vec{A}$ ist nicht eindeutig, Addition einer Konstanten liefert ebenfalls ein gültiges Vektorpotential

$\rightarrow$ Addition von $\underbrace{\dfrac{\mu_0 I_1}{4\pi}\lim_{z_0\to\infty}\ln\dfrac{4z_0^2}{r_0^2}}_{=const.} \qquad \Rightarrow \qquad \boxed{A_z = \frac{-\mu_0 I_0\cos(\omega_1 t)}{4\pi}\ln\frac{x^2+y^2}{r_0^2}}$

b1)

$\Phi_{12} = \iint\limits_a \vec{B}(\vec{r})\,\mathrm{d}\vec{a} = b\int\limits_{d-l/2}^{d+l/2}\mu_0 H_\varphi(r)\,\mathrm{d}r = b\int\limits_{d-l/2}^{d+l/2}\frac{\mu_0 I_1\,\mathrm{d}r}{2\pi r} = \frac{b\mu_0 I_1}{2\pi}\left[\ln r\right]_{r=d-l/2}^{d+l/2}$

$\boxed{\Phi_{12} = \frac{b\mu_0 I_0\cos(\omega_1 t)}{2\pi}\ln\frac{d+l/2}{d-l/2}}$

b2)

$\Phi_{12} = \oint\limits_c \vec{A}\,\mathrm{d}\vec{c} = b(A_z(d-l/2) - A_z(d+l/2)) = \frac{-b\mu_0 I_1}{4\pi}\ln\frac{(d-l/2)^2}{(d+l/2)^2}$

$\boxed{\Phi_{12} = \frac{b\mu_0 I_0\cos(\omega_1 t)}{2\pi}\ln\frac{d+l/2}{d-l/2}}$

c)

$$\boxed{U_i = -\frac{\mathrm{d}\Phi_{12}}{\mathrm{d}t} = \frac{b\mu_0 I_0 \omega_1 \sin(\omega_1 t)}{2\pi} \ln \frac{d+l/2}{d-l/2}}$$

d)

$$\boxed{L_{12} = \frac{\Phi_{12}}{I_1} = \frac{b\mu_0}{2\pi} \ln \frac{d+l/2}{d-l/2}}$$

e)

$$d = v_0 t \qquad U_i = -\frac{\mathrm{d}\Phi_{12}(x,t)}{\mathrm{d}t} = \frac{-\partial\Phi_{12}}{\partial t} - \frac{\partial\Phi_{12}}{\partial x} \cdot \frac{\partial x}{\partial t} = \frac{b\mu_0 I_0 \omega_1 \sin(\omega_1 t)}{2\pi} \ln \frac{v_0 t+l/2}{v_0 t-l/2} - \frac{b v_0 \mu_0 I_0 \cos(\omega_1 t)}{2\pi} \left(\frac{1}{v_0 t+l/2} - \frac{1}{v_0 t-l/2}\right)$$

$$\boxed{U_i = \frac{b\mu_0 I_0}{2\pi} \left(\omega_1 \sin(\omega_1 t) \ln \frac{v_0 t+l/2}{v_0 t-l/2} + \frac{v_0 l \cos(\omega_1 t)}{(v_0 t)^2-(l/2)^2}\right)}$$

f1)

$$\Phi_{12} = \oint_c \vec{A}\,\mathrm{d}\vec{c} = b(A_z(r_1) - A_z(r_2))$$

$$\boxed{\Phi_{12} = \frac{b\mu_0 I_0 \cos(\omega_1 t)}{4\pi} \ln \frac{d^2+(l/2)^2+dl\cos(\omega_2 t)}{d^2+(l/2)^2-dl\cos(\omega_2 t)}} \quad \Rightarrow \boxed{C_1 = \frac{b\mu_0 I_0}{4\pi}} \qquad \boxed{C_2 = \frac{dl}{d^2+(l/2)^2}}$$

f2)

$$U_i = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} = C_1\omega_1 \sin(\omega_1 t) \ln \frac{1+C_2\cos(\omega_2 t)}{1-C_2\cos(\omega_2 t)} + C_1 \cos(\omega_1 t) \left(\frac{\omega_2 C_2 \sin(\omega_2 t)}{1-C_2\cos(\omega_2 t)} + \frac{\omega_2 C_2 \sin(\omega_2 t)}{1+C_2\cos(\omega_2 t)}\right)$$

$$\boxed{U_i = C_1 \left(\omega_1 \sin(\omega_1 t) \ln \frac{1+C_2\cos(\omega_2 t)}{1-C_2\cos(\omega_2 t)} + \frac{2C_2\omega_2 \cos(\omega_1 t)\sin(\omega_2 t)}{1-C_2^2\cos^2(\omega_2 t)}\right)}$$

Aufgabe 49

a)

<u>für $0 \le t \le T = \frac{q}{v}$ gilt:</u>

$$\Phi = B \int_0^{vt} l(1 + \tfrac{1}{a}(x - vt))\,\mathrm{d}x = Bl\left[x + \tfrac{1}{a}(\tfrac{x^2}{2} - vtx)\right]_{x=0}^{vt} = Bl(vt + \tfrac{1}{a}(\tfrac{1}{2}v^2t^2 - v^2t^2))$$

$$\boxed{\Phi = Blvt(1 - \tfrac{vt}{2a})}$$

b)

$$\boxed{U_i = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} = -Blv(1 - \tfrac{vt}{a})}$$

c)

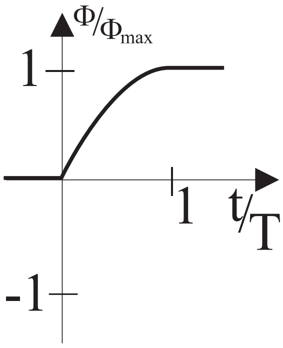
$$\vec{E}_i = \vec{v}\times\vec{B} \qquad U_i = \int_s \vec{E}_i\,\mathrm{d}\vec{s} = \int_s (\vec{v}\times\vec{B})\,\mathrm{d}\vec{s} = -vBl + vB\frac{lvt}{a} \boxed{U_i = -Blv(1 - \tfrac{vt}{a})}$$

f)

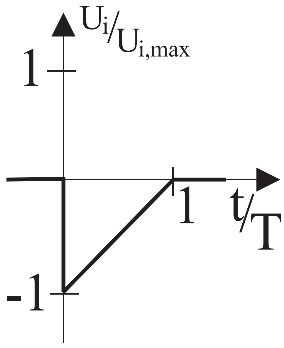
$$U_i = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}(B\tfrac{la}{2}\cos(\omega t)) \qquad \boxed{U_i = \frac{Bla\omega}{2}\sin(\omega t)}$$

zu Aufgabe 49

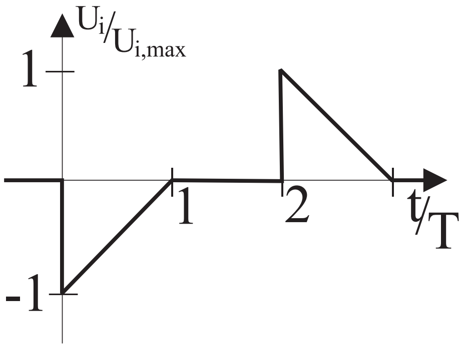
a)

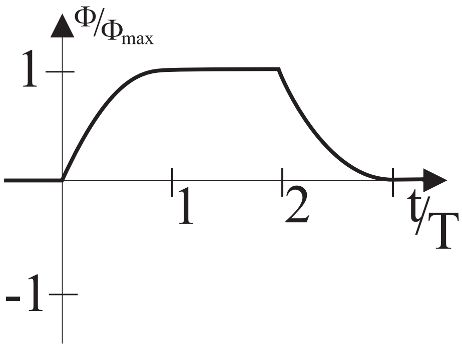


b)



d)





e)