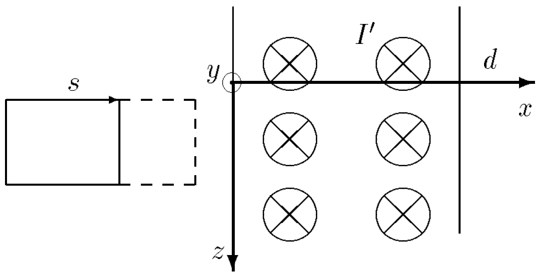
Theorie der Felder und Wellen 2	Name:
Lösungen zum Blatt 18	Matrikelnummer:

Aufgabe 51

a)

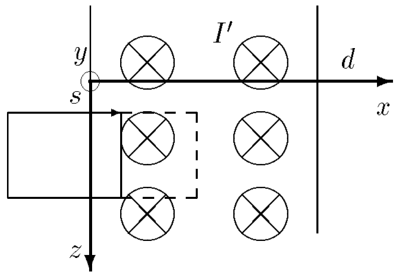
Sei $I' = \frac{I}{h}$ der Strombelag an der Oberfläche der Wände, d. h. der Strom I je Längeneinheit h.
Durchflutungsgesetz: $\oint\limits_s \vec{H}\,\mathrm{d}\vec{s} = \iint\limits_a \vec{J}\,\mathrm{d}\vec{a}$

I) $x < 0$:



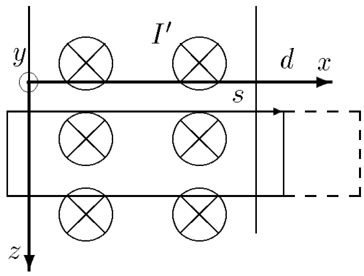
$\vec{H} = H_z \vec{e}_z$, da $\vec{B}$ nach Biot-Savart in Richtung $(\vec{r} - \vec{r}') \times \mathrm{d}\vec{s}$ orientiert ist.
$\iint\limits_a \vec{J}\,\mathrm{d}\vec{a} = 0 \quad \rightsquigarrow \quad \oint\limits_s \vec{H}\,\mathrm{d}\vec{s} = 0$
$\Rightarrow \quad H_z = const.$
Da aber auch $B_z = 0$ für $x \to \infty$ gelten muss, folgt
$\Rightarrow \quad \boxed{H_z = 0} \qquad \boxed{B_z = 0} \qquad \forall x < 0$

II) $0 < x < d$:



$\iint\limits_a \vec{J}\,\mathrm{d}\vec{a} = I\frac{x}{d} = \oint\limits_s \vec{H}\,\mathrm{d}\vec{s} = H_z h$
$\Rightarrow \quad \boxed{H_z = I'\frac{x}{d}} \qquad \boxed{B_z = \mu I'\frac{x}{d}}$

III) $d < x$:



$\iint\limits_a \vec{J}\,\mathrm{d}\vec{a} = I = \oint\limits_s \vec{H}\,\mathrm{d}\vec{s} = H_z h$
$\Rightarrow \quad \boxed{H_z = I'} \qquad \boxed{B_z = \mu I'}$

b)

$U_i = \frac{-\mathrm{d}\Phi}{\mathrm{d}t} = \frac{-\mathrm{d}}{\mathrm{d}t}\iint\limits_a \vec{B}\,\mathrm{d}\vec{a} = \frac{-\mathrm{d}}{\mathrm{d}t}(\mu_0 b(l_0 - vt)I')$
$U_i = \mu_0 b(-l_0\frac{\mathrm{d}I'}{\mathrm{d}t} + vI' + vt\frac{\mathrm{d}I'}{\mathrm{d}t})$
$\boxed{E_y = -\frac{U_i}{b} = -\mu_0(-l_0\frac{\mathrm{d}I'}{\mathrm{d}t} + vI' + vt\frac{\mathrm{d}I'}{\mathrm{d}t})}$

c)

$J_y = \kappa E_y = \frac{-I'}{d} \qquad \rightsquigarrow \frac{-I'}{\kappa d} - \mu_0 l_0\frac{\mathrm{d}I'}{\mathrm{d}t} + \mu_0 vI' + \mu_0 vt\frac{\mathrm{d}I'}{\mathrm{d}t} = 0$
$\boxed{I'(\frac{-1}{\kappa d} + \mu_0 v) + \frac{\mathrm{d}I'}{\mathrm{d}t}(\mu_0 vt - \mu_0 l_0) = 0}$

d)

$v \to 0 \qquad \rightsquigarrow \qquad I'\frac{1}{\kappa d} + \mu_0 l_0 \frac{\mathrm{d}I'}{\mathrm{d}t} = 0$

$\Rightarrow \qquad \boxed{I' = I_0 e^{\frac{-t}{d\kappa\mu_0 l_0}}}$

Exponentiell abklingender Strom ist plausibel.
Die Energie aus dem Magnetfeld im Gebiet 3 fällt als Verlustleistung am ohmschen Widerstand des Blockes ab.

e)

$I(t) = I_0 = const. \qquad \rightsquigarrow \qquad \frac{\mathrm{d}I'}{\mathrm{d}t} = 0 \qquad \Rightarrow \qquad \boxed{v_0 = \frac{1}{\kappa\mu_0 d}}$

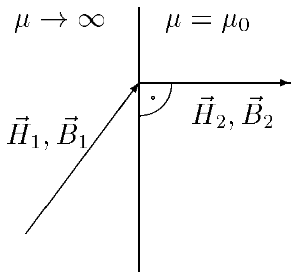
f)

I) $v < v_0$: Strom wird monoton kleiner (langsamer als in d)).

II) $v > v_0$: Strom wird monoton größer.

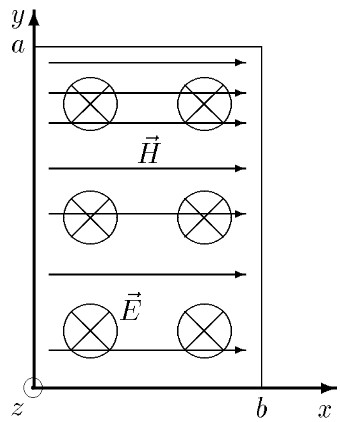
Aufgabe 52

a)



$B_{1n} = B_{2n} = B_2$
$B_{2n} = \mu_0 H_{2n}$
$B_{2t} = 0$
$H_1 = 0$

b)

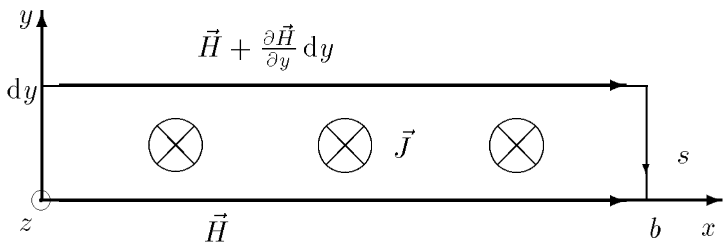


$\vec{H} = H_x \vec{e}_x$
$\vec{E} = E_z \vec{e}_z \qquad (E_z < 0)$

c)

$b\frac{\partial H_x}{\partial y}\,\mathrm{d}y = -J_z b\,\mathrm{d}y = -\kappa E_z b\,\mathrm{d}y$

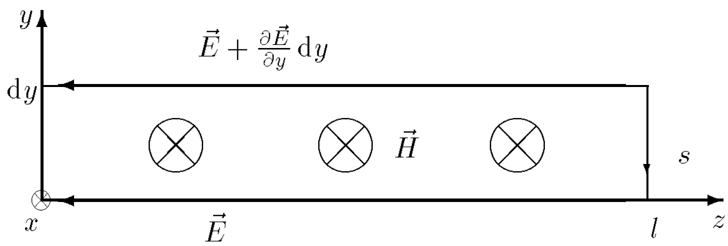
$\boxed{\frac{\partial H_x}{\partial y} = -\kappa E_z}$



d)

$l\frac{\partial E_z}{\partial y}\,\mathrm{d}y = -\frac{\partial}{\partial t}(\mu_0 l H_x\,\mathrm{d}y)$

$\boxed{\frac{\partial E_z}{\partial y} = -\mu_0 \frac{\partial H_x}{\partial t}}$



e)

$\vec{J} = \kappa \vec{E} = \text{ rot } \vec{H} = -\frac{\partial H_x}{\partial y}\vec{e}_z \qquad \Rightarrow \qquad \boxed{\frac{\partial H_x}{\partial y} = -\kappa E_z}$

$\text{rot } \vec{E} = \frac{\partial E_z}{\partial y}\vec{e}_x = -\frac{\partial \vec{B}}{\partial t} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \qquad \Rightarrow \qquad \boxed{\frac{\partial E_z}{\partial y} = -\mu_0 \frac{\partial H_x}{\partial t}}$

f)

$\frac{\partial E_z}{\partial y} = -\mu_0 \frac{\partial H_x}{\partial t} = -\mu_0 \frac{\partial H_x}{\partial y}\frac{\partial y}{\partial t} = \mu_0 \kappa \frac{\partial y}{\partial t} E_z \qquad \rightsquigarrow \qquad \frac{\partial^2 E_z}{\partial y^2} = \mu_0 \kappa \frac{\partial y}{\partial t}\frac{\partial E_z}{\partial y}$

$\boxed{\frac{\partial^2 E_z}{\partial y^2} = \mu_0 \kappa \frac{\partial E_z}{\partial t}}$

$\frac{\partial H_x}{\partial y} = -\kappa E_z \qquad \rightsquigarrow \qquad \frac{\partial^2 H_x}{\partial y^2} = -\kappa \frac{\partial E_z}{\partial y}$

$\boxed{\frac{\partial^2 H_x}{\partial y^2} = \mu_0 \kappa \frac{\partial H_x}{\partial t}}$

g)

$\boxed{\frac{\partial^2 E_z}{\partial y^2} = i\omega\mu_0\kappa E_z} \qquad \boxed{\frac{\partial^2 H_x}{\partial y^2} = i\omega\mu_0\kappa H_x} \qquad (\vec{E}, \vec{H} \in \mathbb{C}^3)$

h)

$\frac{\partial^2 F}{\partial y^2} = i\omega\mu_0\kappa F$

Ansatz: $\qquad F = C_1 e^{ky} + C_2 e^{-ky} \qquad \rightsquigarrow \qquad \frac{\partial^2 F}{\partial y^2} = C_1 k^2 e^{ky} + C_2 k^2 e^{-ky} \stackrel{!}{=} i\omega\mu_0\kappa(C_1 e^{ky} + C_2 e^{-ky})$

$\Rightarrow \qquad k^2 = i\omega\kappa\mu_0 \qquad \Leftrightarrow \qquad \boxed{k = \pm \frac{1+i}{\sqrt{2}}\sqrt{\omega\kappa\mu_0}}$

i)

$H_x = C_1 e^{ky} + C_2 e^{-ky} \qquad \rightsquigarrow \qquad H_x|_{y=0} = C_1 + C_2 \stackrel{!}{=} 0 \qquad \Rightarrow \qquad \boxed{C_1 = -C_2 =: H_0}$

$H_x|_{y=a} = H_0(e^{ka} - e^{-ka}) = 2H_0 \sinh(ka) \stackrel{!}{=} \frac{-I_z}{b} \qquad \Rightarrow \qquad \boxed{H_0 = \frac{-I_z}{2b\sinh(ka)}}$

$\boxed{H_x = 2H_0 \sinh(ky)}$

j)

$J_z = \kappa E_z = \frac{-\partial H_x}{\partial y} = -2H_0 \frac{\partial \sinh(ky)}{\partial y}$

$\boxed{J_z = -2H_0 k \cosh(ky)}$