Radio Frequency Technology III

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Jochen Bauknecht
Andreas Selig

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aditional supplement pages:
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Dieses Dokument wurde mit LaTeX gesetzt.

1 Antennas

1.1 Fundamental Parameters

- Wavelength

$$\boxed{\lambda = \frac{c}{f} = \frac{3*10^8\,\frac{\mathrm{m}}{\mathrm{s}}}{f} = \frac{\lambda_c}{\sqrt{\epsilon_r}}} \tag{1}$$

- Radiated Power

$$\boxed{P_t = \int \vec{S} \cdot d\vec{A}} \qquad \boxed{P_{\max} = S \cdot A_e = \iint\limits_a S_{\max} \cdot C^2(\varphi, \vartheta) d\vec{A}} \tag{2}$$

bei Kugel: $d\vec{A} = r^2 \sin\vartheta \, d\varphi \, d\vartheta$ $\boxed{P_{mW} = 10^{\frac{P_{dBm}}{10}}}$

- Gain

$$\boxed{G = \frac{S_{\max}}{S_I}|_{P_{t0}=\mathrm{const.}}} \tag{3}$$

- Power Density

$$\boxed{S = \frac{|E_0^2|}{2Z_{F0}}} \qquad Z_{F0} = 120\pi\Omega \approx 377\Omega \tag{4}$$

$$\boxed{S_I = \frac{P_{t_0}}{4\pi r^2}} \tag{5}$$

- Directivity

$$\boxed{D = \frac{S_{\max}}{S_I}|_{P_t=\mathrm{const.}} = \frac{G}{\eta}} \tag{6}$$

- Radiation Efficiency

$$\boxed{\eta = \frac{P_t}{P_{t0}} = \frac{R_r}{R_r+R_p}} \qquad \boxed{R_D = \frac{X_L}{Q_L}} \tag{7}$$

- Effective Area

$$\boxed{A_e = \frac{P_{r\max}}{S} = \frac{\lambda^2}{4\pi} \cdot G = A_0 \cdot \eta = \frac{l_e^2 Z_{F0}}{4R_a}} \tag{8}$$

- Maximum effective Area

$$\boxed{A_0 = \frac{P_{r0\max}}{S} = \frac{\lambda^2}{4\pi} \cdot D = \frac{A_e}{\eta} = \frac{2P_{r0\,\max} Z_{F0}}{|E_0^2|} = \frac{2P_{r0\,\max} Z_{F0} l_e^2}{U_0^2}} \tag{9}$$

1.2 Wireless Transmission in Free Space

- Friis' Equation

$$\boxed{P_{r\max} = P_{t0} \cdot G_1 \cdot G_2 \cdot \left(\frac{\lambda_0}{4\pi r}\right)^2 = S \cdot A_e} \tag{10}$$

- Radiation Link Attenuation

$$\boxed{\frac{a}{\mathrm{dB}} = -10\lg\left(\frac{P_{r\max}}{P_{t0}}\right) = 22 + 20\lg\left(\frac{r}{\lambda_0}\right) - g_1 - g_2} \tag{11}$$

- Isotropic Radio Link Attenuation

$$\boxed{\frac{a_0}{\mathrm{dB}} = 22 + 20\lg\left(\frac{r}{\lambda_0}\right)} \tag{12}$$

1.2.1 Hertzian Vectors

- in general

$$\boxed{\vec{\pi} = \frac{1}{j\omega\epsilon_0 4\pi} \iiint\limits_{V'} \vec{J}(\vec{r'}) \frac{e^{-j\beta_0|\vec{r}-\vec{r'}|}}{|\vec{r}-\vec{r'}|} dV'} \tag{13}$$

$$\boxed{\vec{\pi_m} = \frac{1}{j\omega\mu_0 4\pi} \iiint\limits_{V'} \vec{M}(\vec{r'}) \frac{e^{-j\beta_0|\vec{r}-\vec{r'}|}}{|\vec{r}-\vec{r'}|} dV'} \tag{14}$$

- Farfield Approximation

$$\boxed{\vec{\pi} = \frac{e^{-j\beta_0 r}}{j\omega\epsilon_0 4\pi r} \iiint\limits_{V'} \vec{J}(\vec{r'}) e^{j\beta_0 \frac{\vec{r'}\vec{r}}{r}} dV'} \tag{15}$$

$$\boxed{\vec{\pi_m} = \frac{e^{-j\beta_0 r}}{j\omega\mu_0 4\pi r} \iiint\limits_{V'} \vec{M}(\vec{r'}) e^{j\beta_0 \frac{\vec{r'}\vec{r}}{r}} dV'} \tag{16}$$

1.3 Wireless Transmission over Perfectly Reflecting Ground

- Maximum received Power

$$\boxed{P_{r\max} = \frac{G_t \cdot G_r \cdot (h_t \cdot h_r)^2}{r^4}} \tag{17}$$

- Break Point Distance

$$\boxed{r_{bk} \approx 8.4 \frac{h_t \cdot h_r}{\lambda_0}} \tag{18}$$

1.4 Rectangular Apertures

- Aperture with Uniform Field Distribution (cos field dist.: see Horn Antenna)

$$\boxed{E_\vartheta = \frac{A \cdot B \cdot e^{-j\beta_0 r}}{j \cdot \lambda_0 \cdot r} E_0 \cdot C(\varphi, \vartheta)} \tag{19}$$

$$\boxed{C(\varphi, \vartheta) = \text{ si } \left(\pi \frac{A}{\lambda_0} \sin\vartheta \sin\varphi\right) \cdot \text{ si } \left(\pi \frac{B}{\lambda_0} \cos\vartheta\right) \cdot \cos\varphi} \tag{20}$$

The $\cos\varphi$ will be neglected for main beam and first site losses

- H-Plan (xy-plane $\hat{=} \vartheta = \frac{\pi}{2}$)

$$\boxed{C_H(\varphi) = \text{ si } \left(\pi \frac{A}{\lambda_0} \sin\varphi\right)} \tag{21}$$

- E-Plan (xz-plane $\hat{=} \varphi = 0$)

$$\boxed{C_E(\vartheta) = \text{ si } \left(\pi \frac{B}{\lambda_0} \sin\vartheta\right)} \tag{22}$$

- HPBW $B \gg \lambda$

$$\boxed{\frac{\vartheta_{B3dB}}{deg} \approx 50,4 \frac{\lambda_0}{B}} \tag{23}$$

- Directivity

$$\boxed{D = 4\pi \frac{A_0}{\lambda_0^2} \approx \frac{41252}{\frac{\vartheta_{3db}}{\text{deg}} \frac{\varphi_{3db}}{\text{deg}}} \approx \frac{4\pi}{\vartheta_{3db}\varphi_{3db}} (\text{bogenmaß})} \tag{24}$$

- Maximum Effective Area

$$\boxed{A_0 = \frac{\lambda_0^2}{4\pi} D} \tag{25}$$

1.4.1 Equivalent Sources

- electric surface currents

$$\boxed{\vec{J}_* = \hat{n} \times \vec{H}(0)} \tag{26}$$

- magnetic surface currents

$$\boxed{\vec{M}_* = -\hat{n} \times \vec{E}(0)} \tag{27}$$

- if $\kappa \to \infty$ is replaced by $\kappa = 0$ (image theory)

$$\boxed{\vec{M}_*' = 2\vec{M}_*} \tag{28}$$

or

$$\boxed{\vec{J}_*' = 2\vec{J}_*} \tag{29}$$

1.4.2 Patch Antenna

- resonance frequnency

$$\boxed{f_R = \frac{c_0}{2 \cdot l \cdot \sqrt{\epsilon_r}}} \tag{30}$$

- nur die Speisekante und die der Speisekante gegenüberliegende Kante strahlen

1.5 Horn Antenna

- Feld

$$\boxed{|E_0| = \sqrt{\frac{4Z_{FH}P_t}{AB}}} \tag{31}$$

- Resistance

$$\boxed{Z_{FH} = \frac{Z_{F0}}{\sqrt{1-(\frac{\lambda_0}{2A})^2}}} \tag{32}$$

A,B sind die Abmessungen

- Horn of infinite Length
  - Feld

$$\boxed{E_\vartheta = H_\varphi Z_{F0} = \frac{2}{\pi}\frac{E_0 e^{-j\beta_0 r}AB}{j\lambda_0 r}C(\vartheta,\varphi)} \tag{33}$$

  - Distribution (cos field distribution)

$$\boxed{C(\vartheta,\varphi) = |\cos\varphi\frac{\cos(\pi\frac{A}{\lambda_0}\sin\varphi\sin\vartheta)}{1-(2\frac{A}{\lambda_0}\sin\varphi\sin\vartheta)^2}\,\mathrm{si}\,(\frac{B}{\lambda_0}\cos\vartheta)|} \tag{34}$$

  - H-Plane HPBW

$$\boxed{\frac{\varphi_{3dB}}{deg} \approx \frac{68{,}9\lambda_0}{A}} \tag{35}$$

  - E-Plane HPBW

$$\boxed{\frac{\vartheta_{3dB}}{deg} \approx \frac{50{,}4\lambda_0}{B}} \tag{36}$$

  - Side-lobe attenuation (Nebenzipfeldämpfung)

$$\boxed{????} \tag{37}$$

1.5.1 Pyramidal Horn

- aperture distribution (illumination)

$$\boxed{E_Z = E_0\cos\frac{\pi y}{A}} \tag{38}$$

- aperture efficiency

$$\boxed{q = \frac{A_0}{A_g} = \frac{\pi^2}{8}q_E q_H} \tag{39}$$

1.5.2 Circular Horn

- distribution

$$\boxed{E_y(\rho) = E_0(p + (1-\frac{\rho^2}{R^2})^n)} \tag{40}$$

- efficiency

$$\boxed{q = \frac{(p+\frac{1}{n+1})^2}{p^2+\frac{2p}{n+1}+\frac{1}{2n+1}}} \tag{41}$$

- effective Area

$$\boxed{A_0 = q\pi R^2} \tag{42}$$

- Directivity

$$\boxed{D = A_0\frac{4\pi}{\lambda_0^2} = \frac{4q\pi^2R^2}{\lambda_0^2}} \tag{43}$$

- HPBW

$$\boxed{\frac{\vartheta_{3db}}{\mathrm{deg}} \approx 57\frac{\lambda_0}{2R}} \tag{44}$$

1.6 Wire Antennas

- Field

$$\boxed{J_* = H(0) = \frac{I(z')}{2\rho\pi}} \tag{45}$$

- Current Distribution

$$\boxed{I(z') = I_{\max}\sin(\beta_0 h - \beta_0|z'|)} \qquad \boxed{I_a = I(0)} \tag{46}$$

- Herzian Dipole

$$\boxed{\vec{\Pi} = \frac{I_a \cdot l_e \cdot e^{-j\beta r}}{j\omega\epsilon_0 4\pi r}} \tag{47}$$

1.6.1 Straight-Lined Dipoles and Monopoles of general length

- impedance

$$\boxed{Z_d = \frac{U_a}{I_a} = \frac{2\frac{U_a}{2}}{I_a} = 2Z_M} \tag{48}$$

Z_d : dipole impedance
Z_m : monopole impedance

- input impedance Z_a

$h > \frac{\lambda_0}{4}$	$h < \frac{\lambda_0}{4}$
R_{rm} aus sup. 29 Smithchart: R_m einzeichen und um $h - \frac{\lambda_0}{4}$ drehen $\rightarrow Z_a$ oder: $Z_a = \frac{1+j\frac{Z_L}{R_{rm}}\tan[\beta_0(h-\frac{\lambda_0}{4})]}{1+j\frac{R_{rm}}{Z_L}\tan[\beta_0(h-\frac{\lambda_0}{4})]}$	R_r aus sup. 29 $X_m = -jZ_L\cot(\beta_0 h)$ $Z_a = R_r + X_m$

- characteristic impedance

$$\boxed{\text{dipole: } Z_L \approx 120\Omega(\ln\frac{h}{\rho} - 0.8) \qquad \text{monopole: } Z_L \approx 60\Omega(\ln\frac{h}{\rho} - 0.8)} \tag{49}$$

- effective length

$$\boxed{\text{dipole: } l_e = 2\int_0^h \frac{I(z')}{I_a}\,\mathrm{d}z' = 2\frac{1-\cos(\beta_0 h)}{\beta_0\sin(\beta_0 h)} \qquad \text{monopole: } l_e = \int_0^h \frac{I(z')}{I_a}\,\mathrm{d}z' = \frac{1-\cos(\beta_0 h)}{\beta_0\sin(\beta_0 h)}} \tag{50}$$

- Current Distribution

$$\boxed{I(z') = I_{\max}\sin[\beta_0(h - |z'|)]} \tag{51}$$

- characteristic

$$\boxed{C(\vartheta) = \frac{\cos(\beta_0 h\cos\vartheta) - \cos(\beta_0 h)}{\sin\vartheta(1-\cos(\beta_0 h))}} \qquad \boxed{\beta_0 = \frac{2\pi}{\lambda_0}} \tag{52}$$

- Radiation Resistance

$$\boxed{R_r = \text{ ablesen aus schaubild sup 29}} \tag{53}$$

1.6.2 Halfwave Dipole $h = \frac{\lambda_0}{4}$

- Current Distribution

$$\boxed{I(z') = I_a\cos(\beta_0 z')} \tag{54}$$

- Far Field

$$\boxed{E_\vartheta = j\frac{Z_{F0}}{2\pi}I_a\frac{e^{-j\beta_0 r}}{r}C(\vartheta) \qquad C(\vartheta) = \frac{\cos(\frac{\pi}{2}\cos\vartheta)}{\sin\vartheta}} \tag{55}$$

- Radiated Power

$$\boxed{P_t = \frac{1}{2}|I_a|^2 R_r = \frac{U^2}{8R_r}} \tag{56}$$

- Radiation Resistance

$$\boxed{R_r = \frac{Z_{F0}}{\pi}0,61 \approx 73\Omega \approx Z_d \quad \text{(at resonance)}} \tag{57}$$

$$\boxed{Z_M \approx 36.5\Omega} \tag{58}$$

- Directivity

$$\boxed{D \approx 1,64 \hat{=} 2,15dB} \tag{59}$$

1.6.3 Fullwave Dipole $h = \frac{\lambda_0}{2}$

- Radiated Power

$$\boxed{} \tag{60}$$

1.6.4 Electrically small Antennas, Elementary Dipoles

- $$\boxed{I(z') = I_a \left(1 - \frac{|z'|}{h}\right)} \tag{61}$$

  $l_e = h$

- Farfield (see supp 32)

$$\boxed{} \tag{62}$$

1.6.5 Folded Dipoles

- equivalent radius

$$\boxed{\rho_e = \sqrt{\rho_d}} \tag{63}$$

- $$\boxed{Z_d = \frac{U_a}{2I_1} \qquad Z_{TL} = j \cdot Z_{LL} \tan(\beta_0 h)} \tag{64}$$

- $$\boxed{Z_a = \frac{1}{\frac{1}{4Z_d} + \frac{1}{2Z_{TL}}}} \tag{65}$$

  $h = \frac{\lambda_0}{4} \qquad \rightarrow \qquad Z_{TL} \rightarrow \infty \qquad \rightarrow Z_a = 4Z_d \approx 292\Omega$

1.6.6 conical thick dipoles

$$\boxed{Z_{LC} = 120\Omega \cdot \ln \cot \frac{\vartheta_1}{2}} \tag{66}$$

1.7 Antenna Array

- Phase

$$\boxed{\psi = \frac{2\pi}{\lambda_0} d \cos(\varphi_m)} \tag{67}$$

- Distribution

$$\boxed{C_a = C_s \cdot C_g} \tag{68}$$

$$\boxed{C_g = \left|\frac{\sin \frac{n\delta}{2}}{n \sin \frac{\delta}{2}}\right| \qquad \text{n = 2 :} \quad C_g = \left|\cos \frac{\delta}{2}\right|} \tag{69}$$

C_a	...	overall
C_s	...	element pattern
C_g	...	group
φ_m	...	main radiation direction

- Phasenunterschied in der wellenfront (δ ist eine länge)
  - $\varphi = 0$:Antennen stehen hintereinander

$$\boxed{\delta = \psi - \beta_0 d \cos\varphi \sin\vartheta} \, \vartheta \text{ meist } \frac{\pi}{2} \tag{70}$$

  - $\varphi = 0$:Antennen nebeneinander:

$$\boxed{\delta = \psi - \beta_0 d \sin\varphi \sin\vartheta} \, \vartheta \text{ meist } \frac{\pi}{2} \tag{71}$$

  - formale Berechnung

$$\boxed{\delta = \psi - \beta_0 \cdot |\Delta\vec{r}|} \tag{72}$$

$$\boxed{\Delta\vec{r} = \begin{pmatrix} \sin\vartheta\cos\varphi \\ \sin\vartheta\sin\varphi \\ \cos\vartheta \end{pmatrix} \cdot \vec{r'}} \tag{73}$$

2 Rausch

2.1 Fundamental Parameters

	:	max. Verlustleistung bei min. Rausch $\rightarrow$ Eingang: Noise Matching; Ausgang: Power Matching
F	:	Rauschzahl $F = 10^{\frac{F_{db}}{10}}$ $\quad F_{db} = 10 \lg F$
T	:	Temperatur
k	:	Bolzmannkonstante ($k = 1.38066 \cdot 10^{-23} \frac{\mathrm{Ws}}{\mathrm{K}}$)
g	:	Gain der nachfolgenden Verstärker

- SNR (Signal to Noise Ratio)

$$\boxed{S/N = \frac{P_S}{P_N}} \tag{74}$$

- Noise Level

$$\boxed{\frac{\mathrm{NL}}{\mathrm{dB}} = 10 \lg \frac{P_S}{P_N}} \tag{75}$$

- Linear Time Average

$$\boxed{\overline{u(t)} = \lim_{\Delta t \to \infty} \left\{ \frac{1}{\Delta t} \int_{t_0}^{t_o + \Delta t} u(t) dt \right\}} \tag{76}$$

- Mean sqaure

$$\boxed{\overline{u^2(t)} = \lim_{\Delta t \to \infty} \left\{ \frac{1}{\Delta t} \int_{t_0}^{t_o + \Delta t} u^2(t) dt \right\} > 0} \tag{77}$$

- Root Mean Square (RMS)

$$\boxed{\overline{u} = \sqrt{\overline{u^2}}} \tag{78}$$

- Gaussian PDF

$$\boxed{p^*(u_0) = \frac{1}{\sqrt{2\pi \overline{u^2}}} \cdot \exp\left\{ -\frac{1}{2} \left(\frac{u_0}{\sqrt{\overline{u^2}}} \right) \right\}} \tag{79}$$

$$\boxed{P^*(u_1) = \int_{u_0=0}^{u_1} p^*(u_0) du_0 = \Phi^*(\tfrac{u_1}{\overline{u}})} \tag{80}$$

- Voltage Spectral Density

$$\boxed{\lim_{\Delta f \to \infty} \left\{ \frac{\overline{u^2}}{\Delta f} = W_u \right\}} \tag{81}$$

$W_u = \text{const} \rightarrow$ white noise

2.2 Superposition of Noise Signals

- Fully Corellated

$$\boxed{\tilde{U}_s = \tilde{U}_1 + \tilde{U}_2} \tag{82}$$

- Uncorellated

$$\boxed{|\tilde{U}_s| = \sqrt{|\tilde{U}_1|^2 + |\tilde{U}_2|^2}} \tag{83}$$

- Partially Corellated

$$\boxed{|\tilde{U}_s| = \sqrt{|(\tilde{U}_{1C} + \tilde{U}_2|^2 + |\tilde{U}_{1U}|^2}} \boxed{|\tilde{U}_1| = \sqrt{|\tilde{U}_{1U}|^2 + |\tilde{U}_{1C}|^2}} \tag{84}$$

- Corellation Factor

$$\text{uncorrelated} \boxed{\gamma = 0} \qquad \text{correlated} \boxed{\gamma = e^{-j(\varphi_1 - \varphi_2)}} \qquad \text{particular correlated} \boxed{|\gamma| < 1} \tag{85}$$

2.2.1 Wiener Khinchin

$$\boxed{\rho(\tau) = \overline{u(t)u(t+\tau)} = \lim_{\Delta\tau\to\infty}\left\{\frac{1}{\Delta\tau}\int_{-\frac{\Delta\tau}{2}}^{+\frac{\Delta\tau}{2}} u(t)u(t+\tau)dt\right\}} \tag{86}$$

$$\boxed{W_U(f) = 2\int_{-\infty}^{\infty}\rho(\tau)\cos(2\pi f\tau)d\tau} \tag{87}$$

$$\boxed{\rho(\tau) = \int_0^{\infty} W_U(f)\cos(2\pi f\tau)df} \tag{88}$$

2.3 Transmission over linear Network

$$\boxed{W_{u2}(f) = |A(f)|^2 \cdot W_{u1}(f)} \tag{89}$$

2.4 Circles of Constant Noise Figure

- $$\boxed{s = \frac{Z_{\text{opt}} - Z_L}{Z_{\text{opt}} + Z_L} \qquad |\rho| = \sqrt{\frac{F_z - F_{z\,\min}}{F_z + 4\Delta F - F_{z\,\min}}} = \sqrt{1-p}} \tag{90}$$

- Mittelpunkt

$$\boxed{\sigma_i = \frac{|s|(1-|\rho|^2)}{1-|s\rho|^2}}\text{ im "normalen" smith chart} \tag{91}$$

- Radius

$$\boxed{\tau_i = \frac{|\rho|(1-|s|^2)}{1-|s\rho|^2}} \tag{92}$$

2.5 Circles of Constant Mismatch

- Abstand vom Ursprung in Richtung S_{11}^*

$$\boxed{\sigma_i = \frac{|S_{11}|(1-|\rho_i|^2)}{1-|S_{11}\rho_i|^2}}\text{ im "normalen" smith chart} \tag{93}$$

- Radius

$$\boxed{\tau_i = \frac{|\rho_i|(1-|S_{11}\rho_i|^2)}{1-|S_{11}\rho_i|^2}}\ \rho_i\text{ ist die Fehlanpassung als lin. Wert} \tag{94}$$

- S_{11} in der Relexionsebene einzeichnen!!

2.6 Noise of Twopole Network

$$\boxed{W_{ti} = \frac{1}{R^2}W_{tu}} \tag{95}$$

$$\boxed{W_{tu} = 4kRT} \tag{96}$$

- Available Noise Power

$$\boxed{P_{n\,\max} = kT\Delta f} \tag{97}$$

- Effective Bandwidth

$$\boxed{\Delta f = \int_0^{\infty}\frac{W_{ut}}{W_u}df} \tag{98}$$

2.7 Noise of Twoport Network

- Noise

$$\boxed{P = FT\Delta fkg = (T_0 + T_z) \cdot \Delta fkg)} \tag{99}$$

- Noise Figure

$$\boxed{F = 1 + F_z \qquad F_z = F_{z\,\text{min}} + \Delta F \tfrac{4|\rho|^2}{1-|\rho|^2} \qquad \Delta F = (F_z - F_{z\,\text{min}}) \tfrac{1-|\rho|^2}{4|\rho|^2}} \tag{100}$$

- Noise Mismatch

$$\boxed{\rho = \tfrac{Z_i - Z_{\text{opt}}}{Z_i + Z^*_{\text{opt}}}} \tag{101}$$

Z_i : siehts am eingang $F = 10^{\frac{F_{db}}{10}} F_d b = 10 \lg F$
Z_{opt} : möchts am eingang sehen

- Cascading of Twoports

$$\boxed{F_{z\,\text{total}} = F_{z_1} + \tfrac{Fz_2}{g_{T_1\,\text{max}}}} \tag{102}$$

3 RF-Twoport Amplifiers

3.1 Stability

3.1.1 Stability Circles

in der Refexionsebene eintragen! Koordinatenursprung ist meistens Stabil.

- load side
  - Mittelpunkt

$$\boxed{\sigma_l = \tfrac{(S_{22} - \det S \cdot S^*_{11})^*}{|S_{22}|^2 - |\det S|^2}} \tag{103}$$

  - Radius

$$\boxed{\tau_l = \left| \tfrac{S_{12} \cdot S_{21}}{|S_{22}|^2 - |\det S|^2} \right|} \tag{104}$$

- generator side
  - Mittelpunkt

$$\boxed{\sigma_i = \tfrac{(S_{11} - \det S \cdot S^*_{22})^*}{|S_{11}|^2 - |\det S|^2}} \tag{105}$$

  - Radius

$$\boxed{\tau_i = \left| \tfrac{S_{21} \cdot S_{12}}{|S_{11}|^2 - |\det S|^2} \right|} \tag{106}$$

3.1.2 Unconditional Stability

Unconditional Stability is given if

$$\boxed{K = \tfrac{1 - |S_{11}|^2 - |S_{22}|^2 + |\det S|^2}{2|S_{12}| \cdot |S_{21}|} > 1} \tag{107}$$

and

$$\boxed{|\det S| < 1} \tag{108}$$

3.1.3 Power Gain

- Transducer Power Gain

$$\boxed{g_T = \tfrac{P_2}{P_{\text{max}}} = \tfrac{|S_{21}|^2 \cdot (1-|r_i|^2)(1-|r_l|^2)}{|(1-S_{11}r_i)(1-S22r_l) - S_{12}S_{21}r_ir_l|^2}} \tag{109}$$

  with

$$\boxed{r_l = \tfrac{Z_l - Z_L}{Z_l + Z_L}} \tag{110}$$

$$\boxed{r_i = \tfrac{Z_i - Z_L}{Z_i + Z_L}} \tag{111}$$

- Unilateral Transducer Power Gain ($S_{12} = 0$)

$$\boxed{g_{Tu\,\max} = \frac{|S_{21}|^2}{(1-|S_{11}|^2)(1-|S_{22}|^2)}} \tag{112}$$

$$\boxed{g_{Tu} = g_{Tu\,\max} \cdot p_i \cdot p_l} \tag{113}$$

with

$$\boxed{p_i = \frac{(1-|r_i|^2)(1-|S_{11}|^2)}{|1-S_{11}r_i|^2}} \tag{114}$$

$$\boxed{p_l = \frac{(1-|r_l|^2)(1-|S_{22}|^2)}{|1-S_{22}r_l|^2}} \tag{115}$$

Error estimation

$$\boxed{\delta = \text{supp } 53} \tag{116}$$

$$\boxed{\frac{1}{(1+\delta)^2} < \frac{g_T}{g_{Tu}} < \frac{1}{(1-\delta)^2}} \tag{117}$$

3.1.4 Power Match

- input

$$\boxed{Z_i = \frac{S_{11}^*+1}{1-S_{11}^*} \cdot Z_L} \qquad r_i = S_{11}^* \tag{118}$$

- output

$$\boxed{Z_l = \frac{S_{22}^*+1}{1-S_{22}^*} \cdot Z_L} \qquad r_l = S_{22}^* \tag{119}$$